DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Test Booklet No. : 21569

Series A

TEST BOOKLET

Paper—II

Part–II (ACCOUNTANCY/STATISTICS/MATHEMATICS)

(Objective Type)

Time Allowed : 2 Hours **Full Marks : 100**

Read the following instructions carefully before you begin to answer the questions :

1. The name of the Subject, Roll Number as mentioned in the Admission Certificate, Test Booklet No. and Series are to be written legibly and correctly in the space provided on the Answer-Sheet with Black/Blue ballpoint pen.
2. **Answer-Sheet without marking Series as mentioned above in the space provided for in the Answer-Sheet shall not be evaluated.**
3. All questions carry equal marks.

   The Answer-Sheet should be submitted to the Invigilator.

   Directions for giving the answers : Directions for answering questions have already been issued to the respective candidates in the 'Instructions for marking in the OMR Answer-Sheet' along with the Admit Card and Specimen Copy of the OMR Answer-Sheet.

   Example :

   Suppose the following question is asked :

   The capital of Bangladesh is
   (A) Chennai
   (B) London
   (C) Dhaka
   (D) Dhubri

   You will have four alternatives in the Answer-Sheet for your response corresponding to each question of the Test Booklet as below :

   Ⓐ Ⓑ Ⓒ Ⓓ

   In the above illustration, if your chosen response is alternative (C), i.e., Dhaka, then the same should be marked on the Answer-Sheet by blackening the relevant circle with a Black/Blue ballpoint pen only as below :

   Ⓐ Ⓑ ● Ⓓ

   The example shown above is the only correct method of answering.
4. Use of eraser, blade, chemical whitener fluid to rectify any response is prohibited.
5. Please ensure that the Test Booklet has the required number of pages (56) immediately after opening the Booklet. **Students can attend questions of any one subject—Accountancy or Statistics or Mathematics.** In case of any discrepancy, please report the same to the Invigilator.
6. No candidate shall be admitted to the Examination Hall/Room 20 minutes after the commencement of the examination.
7. No candidate shall leave the Examination Hall/Room without prior permission of the Supervisor/ Invigilator. No candidate shall be permitted to hand over his/her Answer-Sheet and leave the Examination Hall/Room before expiry of the full time allotted for each paper.
8. No Mobile Phone, Electronic Communication Device, etc., are allowed to be carried inside the Examination Hall/Room by the candidates. Any Mobile Phone, Electronic Communication Device, etc., found in possession of the candidate inside the Examination Hall/Room, even if on off mode, shall be liable for confiscation.
9. No candidate shall have in his/her possession inside the Examination Hall/Room any book, notebook or loose paper, except his/her Admission Certificate and other connected papers permitted by the Commission.
10. Complete silence must be observed in the Examination Hall/Room. No candidate shall copy from the paper of any other candidate, or permit his/her own paper to be copied, or give, or attempt to give, or obtain, or attempt to obtain irregular assistance of any kind.
11. This Test Booklet can be carried with you after answering the questions in the prescribed Answer-Sheet.
12. Noncompliance with any of the above instructions will render a candidate liable to penalty as may be deemed fit.
13. No rough work is to be done on the OMR Answer-Sheet. You can do the rough work on the space provided in the Test Booklet.

N.B. : There will be negative marking @ 0·25 per 1 (one) mark against each wrong answer.

 [No. of Questions : 100

SEAL

OR

MATHEMATICS

1. Let us consider the sequence

$$\left\{\frac{1}{\log_2(1+n)} : n \in \mathbb{N}\right\}$$

Then the number of terms of the sequence that will be outside the interval $\left(\frac{-1}{10}, \frac{1}{10}\right)$ is

(A) $2^5 - 1$

(B) $2^{10} - 1$

(C) $2^7 + 1$

(D) $2^{11} - 1$

2. Rabin lent ₹ 10,000 to Bimal for 3 years and ₹ 6,000 to Champak for 4 years on simple interest at the same rate of interest and received ₹ 5,400 in all from both of them as interest. What is the rate of interest?

(A) 10%

(B) 12·5%

(C) 15%

(D) 20%

3. Let us consider the function $f:[-1, 1] \to [-1, 1]$ such that $f(x) = |x|$. Then

(A) Rolle's theorem is applicable to f on $[-1, 1]$

(B) the function f has a local minima because $f'(0) = 0$

(C) Rolle's theorem is not applicable to f on $[-1, 1]$ as the function is not continuous on $[0, 1]$

(D) the function f has a local minima but $f'(0)$ does not exist

4. Let $f(x, y) = \sqrt{|xy|}$; $x, y \in \mathbb{R}$. Then

(A) $f_x(0, 0)$ exists but $f_y(0, 0)$ does not exist

(B) $f_x(0, 0)$ does not exist but $f_y(0, 0)$ exists

(C) both $f_x(0, 0)$ and $f_y(0, 0)$ exist

(D) neither $f_x(0, 0)$ nor $f_y(0, 0)$ exists

5. In what ratio rice at ₹ 30 per kg should be mixed with rice at ₹ 45 per kg so that on selling the mixture at ₹ 42 per kg there is a profit of 20%?

(A) 2 : 1

(B) 2 : 3

(C) 5 : 2

(D) 3 : 7

6. If $P + \frac{1}{Q} = 1$ and $Q + \frac{1}{R} = 1$, then what is the value of PQR?

(A) 1·5

(B) 3

(C) 1

(D) −1

7. Let $x_1 = x_2 = 1, \ x_{n+2} = x_{n+1} + x_n$.

Then $\lim_{n \to \infty} \frac{x_{n+1}}{x_n} =$

(A) $\frac{1+\sqrt{5}}{2}$

(B) $\frac{1-\sqrt{5}}{2}$

(C) $\frac{1+\sqrt{5}}{3}$

(D) $\frac{1-\sqrt{5}}{3}$

8. Let

$$H = \left\{ \begin{pmatrix} a-3b+6c \\ 5a+4d \\ b-2c-d \\ 5d \end{pmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

Then the dimension of H is

(A) 1

(B) 2

(C) 3

(D) 4

9. Let $(G_1, *)$ and $(G_2, +)$ be groups. Let $a_1, b_1 \in G_1$ and $a_2, b_2 \in G_2$. $G_1 \times G_2$ becomes a group with respect to the operation

$$(a_1, a_2)(b_1, b_2) = (a_1 * b_1, \ a_2 + b_2)$$

If $a_1 \in G_1$ and $a_2 \in G_2$ have orders n and m respectively, then the element (a_1, a_2) has order

(A) $\gcd(n, m)$

(B) $\text{lcm}(n, m)$

(C) m

(D) n

10. The number of vertices of a polyhedron which has 30 edges and 12 faces is

(A) 11

(B) 20

(C) 22

(D) 25

11. Given that $f(3) = 4$, $f'(3) = -\frac{1}{2}$, $f''(3) = 12$, then what is the approximate value of $f(3 \cdot 2)$?

(A) 5·9

(B) 4·14

(C) 6·7

(D) 6·36

12. The angle between the lines $x^2 + xy - 6y^2 = 0$ is

(A) 120°

(B) 90°

(C) 60°

(D) 135°

13. What is the unit digit of $1! + 2! + \ldots + 99! + 100!$?

(A) 3

(B) 1

(C) 5

(D) 6

14. Two taps can fill a tank with water in 15 hours and 12 hours respectively, and a third tap can empty it in 4 hours. If the taps are opened in order at 8 AM, 9 AM and 11 AM respectively, the tank will be emptied at

(A) 11:40 AM

(B) 12:40 PM

(C) 01:40 PM

(D) 02:40 PM

15. The orthogonal trajectories of the family of parabolas $y = cx^2$ are

(A) $x^2 + 2y^3 = k^2$, k is an arbitrary constant

(B) $x^2 + 2y^2 = k^2$, k is an arbitrary constant

(C) $x^3 + 2y^2 = k^2$, k is an arbitrary constant

(D) None of the above

16. The general solution of

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = 0$$

is

(A) $y = (c_1 + c_2 x)e^{3x} + c_3 \cos x$

(B) $y = (c_1 + c_2 x)e^{3x} + c_3 \sin x$

(C) $y = (c_1 + c_2 x)e^{x} + c_3 e^{-x}$

(D) $y = (c_1 + c_2 x)e^{3x} + c_3 e^{-2x}$

17. Let us consider the sequence $X = \left\{\frac{1}{3}(1 + (-1)^n) : n \in \mathbb{N}\right\}$. Which of the following is correct?

(A) X is convergent and all its subsequences are convergent

(B) X is convergent and all its subsequences are not convergent

(C) X is not convergent but some of its subsequences are convergent

(D) X does not have any subsequence

18. How many lines can be drawn through 11 points on a circle?

(A) 66

(B) 45

(C) 51

(D) 55

19. Ramesh goes 50 m to the South of his house. Then he turns left and goes another 20 m. Then, turning to the North, he goes 30 m and then starts walking to his house. In which direction is he walking now?

(A) North-West

(B) North

(C) South-East

(D) East

20. Find the wrong number in the following series :

3, 8, 18, 46, 100, 210, 432

(A) 8

(B) 18

(C) 100

(D) 210

21. Let $A_{n\times k}$ and $B_{k\times n}$ be two matrices. Let I represents different identity matrices of suitable orders and

$$L = \begin{pmatrix} I - BA & B \\ 2A - ABA & AB - I \end{pmatrix}$$

Find the correct statement.

(A) $L^2 \neq I$, where I is the identity matrix.

(B) $L^2 = I$, where I is the identity matrix of order $n + k$.

(C) $L^2 = I$, where I is the identity matrix of order n.

(D) $L^2 = I$, where I is the identity matrix of order k.

22. Let A and B be bounded subsets of $\mathbb{R}$. Then

(A) $\sup(A \cup B) = \sup\{\sup A, \sup B\}$

(B) $\sup(A \cup B) < \sup\{\sup A, \sup B\}$

(C) $\sup(A \cap B) = \sup\{\sup A, \sup B\}$

(D) $\sup(A \cap B) = \sup\{\sup A, \sup B^C\}$

23. Which of the following is correct?

(A) A non-abelian group containing five elements exists

(B) A non-abelian group containing four elements exists

(C) A non-abelian group containing three elements exists

(D) A non-abelian group containing six elements exists

24. Suppose a student wants to make up a schedule for a seven-day period during which she will study one subject each day. She is taking four subjects—Mathematics, Physics, Chemistry and Economics. Then the number of schedules that denote at least one day to each subject is

(A) $4^7 - 4(3^7) + 6(2^7)$

(B) $4^7 - 4(3^7) + 6(2^7) - 4$

(C) $4^8 - 4(3^7) + 6(2^7) - 4$

(D) $4^8 - 5(3^7) + 6(2^7) - 4$

25. The equation of the conic which passes through $(-2, 0)$, touches the y-axis at the origin and has its centre at $(1, 1)$ is

(A) $x^2 - 4xy + 2y^2 + 2 = 0$

(B) $x^2 - 4xy^2 + 2y^2 + 2x = 0$

(C) $x^2 - 4xy + 2y^2 + 2x = 0$

(D) $x^3 - 4xy + 2y^2 + 2x = 0$

26. The distance between the pair of lines

$$x^2 + 6xy + 9y^2 - 5x - 15y + 6 = 0$$

is

(A) $\frac{1}{\sqrt{10}}$ unit

(B) $\frac{2}{\sqrt{10}}$ unit

(C) $\frac{1}{\sqrt{5}}$ unit

(D) $\frac{7}{\sqrt{10}}$ units

27. The average of 4, 6, 8 and p is 7, and the average of 9, 7, 5, p and q is 10. What is the value of q?

(A) 12

(B) 15

(C) 18

(D) 19

28. What number should be divided by $\sqrt{0 \cdot 16}$ to give the result as 16 ?

(A) 0·08

(B) 0·25

(C) 6·4

(D) 0·0064

29. If the radius of a circle is increased by 75%, then the circumference will increase by

(A) 25%

(B) 50%

(C) 75%

(D) 100%

30. If $p\%$ of q is 100 and $q\%$ of r is 200, then find the relation between p and r.

(A) $r = \frac{p}{4}$

(B) $r = 4p$

(C) $r = \frac{p}{2}$

(D) $r = 2p$

31. The equation

$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$

represents

(A) a pair of straight lines

(B) a parabola

(C) an ellipse

(D) a hyperbola

32. If any tangent plane to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts a, b, c on coordinate axes, then

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} =$$

(A) $\frac{2}{r^2}$

(B) $\frac{-1}{r^2}$

(C) $\frac{1}{r^3}$

(D) $\frac{1}{r^2}$

33. The centre and radius of the circle $x^2 + y^2 + z^2 = 49, \quad 2x - y + 3z = 14$ are

(A) $(2, -1, 3)$ and $\sqrt{35}$ respectively

(B) $(2, 1, -3)$ and $\sqrt{35}$ respectively

(C) $(2, -1, 3)$ and $\sqrt{53}$ respectively

(D) $(2, -1, -3)$ and $\sqrt{35}$ respectively

34. If PSP' and QSQ' be two perpendicular focal chords of the conic $\frac{l}{r} = 1 + e\cos\theta$ passing through the focus S, then

$$\frac{1}{SP \times SP'} + \frac{1}{SQ \times SQ'} =$$

(A) $\frac{2 - e^2}{l}$

(B) $\frac{2 + e^2}{l^2}$

(C) $\frac{2 - e^2}{l^2}$

(D) $\frac{2 - e^2}{l^3}$

35. The number of subgroups of order 25 of a group of order 75 is

(A) 3

(B) 1

(C) 5

(D) 7

36. $\lim_{z \to 0} \frac{\bar{z}}{z}$, where z is a complex number

(A) equals to 1

(B) equals to -1

(C) equals to 0

(D) does not exist

37. The cube of any number has one of the forms

(A) $9k$, $9k+1$ or $9k+7$

(B) $9k$, $9k+1$ or $9k+8$

(C) $9k$, $9k+2$ or $9k+7$

(D) $9k$, $9k+3$ or $9k+7$

38. Let $f(x, y, z) = x^2 + y^2 + z^2$ and $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$. Then $\vec{\nabla}^2 f =$

(A) 0

(B) 3

(C) 4

(D) 6

39. Which of the following is correct?

(A) 37 divides $75^{36} - 1$

(B) 37 divides $75^{36} - 2$

(C) 37 divides $75^{36} - 3$

(D) 37 divides $75^{36} + 1$

40. If $f(x) = x$ and $P = \left\{\frac{i}{4};\ i = 0, 1, 2, 3\right\}$, then

(A) $U(P, f) = \frac{5}{8}$, $L(P, f) = \frac{3}{7}$

(B) $U(P, f) = \frac{5}{8}$, $L(P, f) = \frac{3}{8}$

(C) $U(P, f) = \frac{5}{7}$, $L(P, f) = \frac{3}{8}$

(D) $U(P, f) = \frac{5}{7}$, $L(P, f) = \frac{3}{7}$

41. The smallest number which, when diminished by 7, is divisible by 12, 16, 18, 21 and 28, is

(A) 1008

(B) 1015

(C) 1022

(D) 1032

42. Find the value of $\left(\sqrt{5}-\frac{1}{\sqrt{5}}\right)^2$.

(A) 5

(B) 3·65

(C) 3·2

(D) $4\sqrt{5}$

43. 2 men and 7 women can do a piece of work in 14 days. 3 men and 8 women can do the same in 11 days. Then, 8 men and 6 women can do three times the amount of this work in

(A) 18 days

(B) 21 days

(C) 24 days

(D) 30 days

44. Let X be a continuous random variable. Let a, b belong to the domain of X, then

(A) $P(X = a) > 0$

(B) $P(a < X < b) = P(a \leq X \leq b)$

(C) $P(X < a) = 1$

(D) $P(X > b) > 1$

45. Let α, β and γ be the orders of convergent of bisection method, Newton-Raphson method and secant method respectively, then

(A) $\alpha > \gamma > \beta$

(B) $\alpha < \beta < \gamma$

(C) $\gamma < \alpha < \beta$

(D) $\alpha < \gamma < \beta$

46. If α, β and γ are the roots of the equation $x^3 + 3x^2 + 2x + 1 = 0$, then $\alpha^3 + \beta^3 + \gamma^3$ is equal to

(A) −12

(B) 12

(C) −21

(D) 21

47. If A and B be two vectors, then $|A \times B|^2$ equals to

(A) $|A|^2 \, |B|^2$

(B) $|A|^2 \, |B|^2 + (A \cdot B)^2$

(C) $|A|^2 \, |B|^2 - (A \cdot B)^2$

(D) $|A|^2 \, |B|^2 - (A \cdot B)$

48. Which of the following is correct?

(A) $\gcd(2, 5) > \gcd(-9, 16) > \gcd(-27, -35)$

(B) $\gcd(2, 5) < \gcd(-9, 16) < \gcd(-27, -35)$

(C) $\gcd(2, 5) = \gcd(-9, 16) = \gcd(-27, -35)$

(D) None of the above

49. Let $F(x, y, z)$ be a scalar-valued function, and $G(x, y, z)$ be a vector-valued function. Let $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$. Then

(A) $\vec{\nabla} \times (\vec{\nabla} F) = 0$ and $\vec{\nabla} \cdot (\vec{\nabla} \times G) = 0$

(B) $\vec{\nabla} \times (\vec{\nabla} \times F) = 0$ and $\vec{\nabla} \cdot (\vec{\nabla} \times G) = 0$

(C) $\vec{\nabla} \times (\vec{\nabla} \times F) = 0$ and $\vec{\nabla} \cdot (\vec{\nabla} \cdot G) = 0$

(D) $\vec{\nabla} \times (\vec{\nabla} \cdot F) = 0$ and $\vec{\nabla} \cdot (\vec{\nabla} \times G) = 0$

50. Let A be an invertible $n \times n$ matrix, say $A = [a_1, a_2, \ldots, a_n]$. Then

(A) the columns of A are orthogonal

(B) the columns of A are linearly independent

(C) the columns of A form a basis for $\mathbb{R}^m$, $m < n$

(D) the columns of A form a basis for $\mathbb{R}^n$

51. A tap can fill a tank in 3 hours. There are two outlet taps from the tank which can empty it in 10 hours and 7 hours respectively. If all 3 taps are opened simultaneously, then the approximate time needed to fill the tank is

(A) 11 hours

(B) 10 hours

(C) 9 hours

(D) 8 hours

52. A man can complete a journey in 10 hours. He travels the first half of the journey at the rate of 21 km/hr and the second half at the rate of 24 km/hr. Find the total journey.

(A) 220 km

(B) 224 km

(C) 230 km

(D) 234 km

53. A rectangular carpet has an area of 120 sq. m and a perimeter of 46 m. The length of its diagonal is

(A) 15 m

(B) 16 m

(C) 17 m

(D) 20 m

54. If A, B and $A+B$ are each non-singular square matrix, then

(A) $A^{-1}(A+B) = B^{-1}(A+B) = (A^{-1}+B^{-1})^{-1}$

(B) $A(A+B)^{-1} = B(A+B)^{-1} = (A^{-1}+B^{-1})^{-1}$

(C) $A(A+B)^{-1}B = B(A+B)^{-1}A = (A^{-1}+B^{-1})^{-1}$

(D) $AB(A+B)^{-1} = BA(A+B)^{-1} = (A^{-1}+B^{-1})^{-1}$

55. Let us consider the equation

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$$

Then, $\theta = y^{1-n}$ can reduce the above equation to

(A) $\frac{dx}{d\theta} = Q(x) + (n^2 - 1)\,P(x)\theta$

(B) $\frac{d\theta}{dx} = Q(x) + (1 - n^2)\,P(x)\,\theta$

(C) $\frac{d\theta}{dx} = Q(x) + (n^2 - 1)\,P(x)\theta$

(D) $\frac{d\theta}{dx} = (1 - n)\,Q(x) + (n - 1)\,P(x)\,\theta$

56. Let y be some approximation of x. Which of the following statements is correct?

(A) Relative error $= \frac{|x - y|}{|x|}$

(B) Relative error $= \frac{|x|}{|x| - |y|}$

(C) Relative error $= \frac{|x|}{|x - y|}$

(D) Relative error $= \frac{100x}{|x - y|}$

57. Let $x_n = k + (-1)^n \frac{1}{n}$; $n \in \mathbb{N}$, k is an integer, then

(A) $\overline{\lim_{n\to\infty}}\, x_n = k$ and $\underline{\lim_{n\to\infty}}\, x_n = 0$

(B) $\overline{\lim_{n\to\infty}}\, x_n = k$ and $\underline{\lim_{n\to\infty}}\, x_n = \infty$

(C) $\overline{\lim_{n\to\infty}}\, x_n = \infty$ and $\underline{\lim_{n\to\infty}}\, x_n = k$

(D) $\overline{\lim_{n\to\infty}}\, x_n = k$ and $\underline{\lim_{n\to\infty}}\, x_n = k$

58. The line $\frac{l}{r} = a\cos\theta + b\sin\theta$ may touch the conic $\frac{l}{r} = 1 + e\cos(\theta - \beta)$ under the condition

(A) $(a + e\cos\beta)^2 + (b - e\sin\beta)^2 = 1$

(B) $(a - e\cos\beta)^2 + (b - e\sin\beta)^2 = 1$

(C) $(a - e\cos\beta)^2 + (b + e\sin\beta)^2 = 1$

(D) $(a - e\cos\beta)^2 + (b - e\sin\beta)^2 = e$

59. The characteristic of the ring $Z[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in Z\}$ is

(A) 0

(B) 1

(C) 7

(D) 11

60. The plane $x = 2$ meets the ellipsoid $\frac{x^2}{16} + \frac{y^2}{12} + \frac{z^2}{4} = 1$ in an ellipse of area

(A) $3\sqrt{3}\,\pi^2$

(B) $3\sqrt{2}\,\pi^2$

(C) $3\sqrt{3}\,\pi$

(D) $3\sqrt{2}\,\pi$

61. $\int_C \bar{z}\,dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by $z = t^2 + it$ is

(A) $10 + \frac{8i}{3}$

(B) $40 + \frac{8i}{3}$

(C) $10 - \frac{8i}{3}$

(D) $1 - \frac{8i}{3}$

62. $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$, where C is the circle $|z| = 3$ equals to

(A) 0

(B) $2\pi i$

(C) $3\pi i$

(D) $4\pi i$

63. Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ be any three vectors. Then find the correct expression.

(A) $(\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})$

(B) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

(C) $(\vec{A} \cdot \vec{B}) \cdot \vec{C} = \vec{A} \cdot (\vec{B} \cdot \vec{C})$

(D) $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \times \vec{C}$

64. If a and b are both odd integers, then

(A) 16 divides $a^4 + b^4 + 1$

(B) 16 divides $a^4 + b^4 - 3$

(C) 16 divides $a^2 + b^2 - 2$

(D) 16 divides $a^4 + b^4 - 2$

65. Which of the following is correct?

(A) $39! \equiv -1 \pmod{41}$

(B) $69! \equiv 2 \pmod{71}$

(C) $35! \equiv 1 \pmod{37}$

(D) None of the above

66. Let α_i, $i = 1, 2, 3, 4, 5$ be the roots of the equation $x^5 - 1 = 0$. Then

$$\sum_{i=1}^{5} \alpha_i^7 =$$

(A) 1

(B) −1

(C) 0

(D) 2

67. The equation $x^3 + qx + r = 0$ has two equal roots, if

(A) $r^2 + q^2 = 0$

(B) $27r^2 + 4q^3 = 0$

(C) $27r^2 - 4q^3 = 0$

(D) $r^3 + 3q^2 = 0$

68. Let us consider the following LPP :

Minimize $Z = -2x_1 - 3x_2$

subject to

$$x_1 + 2x_2 \geq 2$$
$$x_1, x_2 \geq 0$$

Then

(A) (0, 0) is a feasible solution

(B) the optimal solution is unbounded

(C) the optimal solution is a finite quantity

(D) None of the above

69. Let $S = \{x : Ax = b\}$, where A is an $m \times n$ matrix and b is an m-vector. Then

(A) if $x, y \in S$, then $\eta x + (1-\eta) y \in S$, $\forall \eta \in (0, 1)$

(B) if $x \in S$, then $\eta x \in S$, $\forall \eta \in (0, 1)$

(C) $x = 0 \in S$

(D) None of the above

70. If $Q = \{y_0 = 0; y_1, y_2, \ldots, y_m = 1\}$ such that $y_0 < y_1 < \ldots < y_m$, then

(A) $U(P \cap Q, f) \leq U(P, f)$ and $L(P \cup Q, f) \geq U(P, f)$

(B) $U(P \cup Q, f) \leq U(P, f)$ and $L(P \cap Q, f) \geq U(P, f)$

(C) $U(P \cup Q, f) \leq U(P, f)$ and $L(P \cup Q, f) \geq L(P, f)$

(D) $U(P \cap Q, f) \leq U(P, f)$ and $L(P \cap Q, f) \geq U(P, f)$

71. Consider the following permutations in S_7 with the usual operations

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix}$$

and $$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$$

Then $\sigma\tau\sigma^{-1} =$

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 7 & 6 & 4 & 5 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 7 & 6 & 5 & 4 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 3 & 5 & 6 & 2 & 7 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 2 & 7 & 6 & 4 & 5 \end{pmatrix}$

72. Let us consider the function

$$f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}; \quad (x, y) \neq (0, 0)$$

$$f(0, 0) = 0$$

Then

(A) $f_{xy}(0, 0) = -1$ and $f_{yx}(0, 0) = 1$

(B) $f_{xy}(0, 0) = 1$ and $f_{yx}(0, 0) = 1$

(C) $f_{xy}(0, 0) = 1$ and $f_{yx}(0, 0) = -1$

(D) $f_{xy}(0, 0) = -1$ and $f_{yx}(0, 0) = -1$

73. The number of edges of the complement of a cycle graph having 100 vertices (C_{100}) is

(A) 100

(B) 400

(C) 3735

(D) 4850

74. Let $(Z_9, +)$ be the group with operation addition modulo 9. Let $H = \{0, 3, 6\}$. Then, the number of distinct cosets of H in Z_9 is

(A) 2

(B) 3

(C) 4

(D) 5

75. The value of $\lim_{z \to 2e^{i\pi/3}} \frac{(z^3 + 8)}{z^4 + 4z^2 + 16}$ is

(A) $\frac{3}{8} - i\frac{\sqrt{3}}{8}$

(B) $\frac{3}{8} - i\frac{\sqrt{3}}{13}$

(C) $\frac{-3}{8} - i\frac{\sqrt{3}}{8}$

(D) $\frac{3}{8} + i\frac{\sqrt{3}}{13}$

76. Let $a = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$, $b = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ and $c = \begin{pmatrix} 6 \\ 16 \\ -5 \end{pmatrix}$

be elements of the vector space $\mathbb{R}^3$. Let $H = \text{span}\{a, b, c\}$. Then the dimension of H is

(A) 1

(B) 2

(C) 3

(D) 4

77. A tank is 25 m long, 12 m wide and 6 m deep. The cost of painting the four walls and the bottom of the tank at ₹ 0·75 per sq. m is

(A) ₹ 456

(B) ₹ 458

(C) ₹ 568

(D) ₹ 558

78. The price of a shirt is marked 35% above its cost price. The percentage of discount must be allowed to gain 8% is

(A) 27

(B) 25

(C) 20

(D) 18

79. The diameter of a driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 km per hour?

(A) 200

(B) 250

(C) 300

(D) 350

80. In how many different ways can the letters of the word ATREX be arranged so that the vowels are always together?

(A) 120

(B) 72

(C) 24

(D) 48

81. A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to form cones, each of height 1 cm and base radius 1 mm. The number of cones is

(A) 13500

(B) 4500

(C) 1350

(D) 450

82. What must be subtracted from both the terms p and q so that their ratio reverses?

(A) pq

(B) $\sqrt{pq}$

(C) $p+q$

(D) $\sqrt{p+q}$

83. Find the remainder when 1643276571 is divided by 25.

(A) 24

(B) 23

(C) 21

(D) 19

84. If $p = 256$, then the value of

$$(p^{1/8} - 1)(p^{1/8} + 1)$$

is

(A) 3

(B) 16

(C) 0

(D) 128

85. Find the value of

$$\frac{5\cdot3\times9\cdot6+4\cdot7\times9\cdot6}{4\cdot8\times9\cdot7-4\cdot8\times8\cdot7}$$

(A) 4·3

(B) 20

(C) 6·6

(D) 10

86. Find the value of

$$5\frac{7}{9}+7\frac{11}{8}+13\frac{1}{9}-16\frac{1}{3}$$

(A) $8\frac{5}{3}$

(B) $9\frac{5}{9}$

(C) $7\frac{1}{3}$

(D) $\frac{61}{6}$

87. Find the value of

$$\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{7}+\sqrt{8}}$$

(A) $2\sqrt{2}$

(B) $\sqrt{2}$

(C) $\sqrt{3}$

(D) $3\sqrt{2}$

88. Let $A = \begin{pmatrix} 7 & 2 \\ -4 & 1 \end{pmatrix}$. Given that $A = PDP^{-1}$, where $P = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$ and $D = \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix}$, then

(A) $A^k = \begin{pmatrix} 2\cdot 5^k + 3^k & 5^k - 3^k \\ 2\cdot 3^k - 2\cdot 5^k & 2\cdot 3^k - 5^k \end{pmatrix}$

(B) $A^k = \begin{pmatrix} 2\cdot 5^k - 3^k & 5^k + 3^k \\ 2\cdot 3^k - 2\cdot 5^k & 2\cdot 3^k - 5^k \end{pmatrix}$

(C) $A^k = \begin{pmatrix} 2\cdot 5^k - 3^k & 5^k - 3^k \\ 2\cdot 3^k - 2\cdot 5^k & 2\cdot 3^k - 5^k \end{pmatrix}$

(D) $A^k = \begin{pmatrix} 2\cdot 5^k - 3^k & 5^k - 3^k \\ 2\cdot 3^k - 2\cdot 5^k & 2\cdot 3^k + 5^k \end{pmatrix}$

89. By selling an article for ₹816, a shopkeeper incurs a loss of 20%. At what price should he sell to gain 10%?

(A) ₹ 1,220

(B) ₹ 1,170

(C) ₹ 1,153

(D) ₹ 1,122

90. Let $\hat{i}$, $\hat{j}$ be unit vectors along X and Y axes respectively. The unit normal vector for the circular helix $x = a\cos t$, $y = a\sin t$, $z = ct$, $a > 0$ is

(A) $-(\cos t\hat{i} - \sin t\hat{j})$

(B) $-(\cos t\hat{i} + \sin t\hat{j})$

(C) $\cos t\hat{i} + \sin t\hat{j}$

(D) $\cos t\hat{i} - \sin t\hat{j}$

91. The linear Diophantine equation $172x + 20y = 1000$ has

(A) no integer solution

(B) a unique integer solution

(C) infinite integer solutions

(D) finite number of integer solutions

92. Moment of inertia of a uniform rod of mass m and length l about an axis passing through one of the end points of the rod and perpendicular to the rod is

(A) $\frac{m^2 l^2}{2}$

(B) $\frac{4ml^2}{3}$

(C) $\frac{ml^2}{2}$

(D) $\frac{ml^2}{3}$

93. Let X be a continuous random variable whose probability density function $f_X : X \to \mathbb{R}$ such that

$$f_X(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then $P\left(\left\{X < \frac{1}{8}\right\} \cup \left\{X > \frac{7}{8}\right\}\right) =$

(A) $\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

94. The initial value problem

$$\frac{\partial u}{\partial t} + \frac{u \partial u}{\partial x} = x; \; u(x, 0) = 1$$

has the solution

(A) $u = x - 2e^{-t} - (u + x)e^{-2t}$

(B) $u = x + 2e^{-t} - (u + x)e^{-2t}$

(C) $u = x + 2e^{t} - (u + x)e^{-2t}$

(D) $u = x + 2e^{-t} + (u + x)e^{-2t}$

95. Let $y = (1 - 5x + 6x^2)^{-1}$. Let y_n represents the nth derivative of y. Then $y_5 =$

(A) $100\left[\left(\frac{3}{1-3x}\right)^7 - \left(\frac{2}{1-2x}\right)^7\right]$

(B) $100\left[\left(\frac{3}{1-3x}\right)^6 - \left(\frac{2}{1-2x}\right)^6\right]$

(C) $120\left[\left(\frac{3}{1-3x}\right)^5 - \left(\frac{2}{1-2x}\right)^5\right]$

(D) $120\left[\left(\frac{3}{1-3x}\right)^6 - \left(\frac{2}{1-2x}\right)^6\right]$

96. Let G be a group of real numbers under addition and let $\overline{G}$ be the group of the positive real numbers under multiplication. Let $\Phi : G \to \overline{G}$. Then

(A) $G \cong \overline{G}$ under the mapping $\Phi(x) = 2^x$

(B) $G \cong \overline{G}$ under the mapping $\Phi(x) = x$

(C) $G \cong \overline{G}$ under the mapping $\Phi(x) = x^2$

(D) $G \cong \overline{G}$ under the mapping $\Phi(x) = x^3$

97. The function $f(z) = \frac{z}{(z^2+4)^2}$, where z is a complex number, has

(A) two poles of order two and none of them is isolated singularity

(B) two poles of order three and each of them is isolated singularity

(C) two poles of order two and each of them is isolated singularity

(D) three poles of order two and each of them is isolated singularity

98. Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three vectors. Consider the following expressions :

$$P: \vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

$$Q: \vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} + (\vec{A}\cdot\vec{B})\vec{C}$$

Find the correct statement from the following.

(A) P is correct but Q is wrong.

(B) P is wrong and Q is wrong.

(C) P is wrong but Q is correct.

(D) P is correct and Q is correct.

99. Let $p_1, p_2, \ldots, p_n$ be successive prime numbers and $p_1 = 2$, then

(A) $(p_1 p_2 \cdots p_n)+1$ is prime but $(p_1 p_2 \cdots p_n)-1$ is composite

(B) $(p_1 p_2 \cdots p_n)-1$ is prime but $(p_1 p_2 \cdots p_n)+1$ is composite

(C) $(p_1 p_2 \cdots p_n)\pm 1$ are composites

(D) $(p_1 p_2 \cdots p_n)\pm 1$ are prime numbers

100. Let the random variable X has probability density function

$$f_X(x) = \begin{cases} \frac{(x+1)}{2} & , \quad -1 < x < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Then the mean and variance of X are

(A) $\frac{1}{5}$ and $\frac{2}{9}$ respectively

(B) $\frac{1}{5}$ and $\frac{2}{7}$ respectively

(C) $\frac{1}{3}$ and $\frac{2}{9}$ respectively

(D) $\frac{1}{3}$ and $\frac{2}{13}$ respectively

SPACE FOR ROUGH WORK

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